NOTATION

v	is the velocity of fluid;
ω	is the intrinsic angular velocity of fluid;
ρ	is the density;
p	is the pressure;
t	is the time;
T	is the temperature;
g	is the gravitational acceleration;
c_p	is the isotropic specific heat;
θ	is the thermal conductivity;
J	is the scalar constant with dimensions of moment of inertia per unit mass;
ω	is the cyclic frequency;
K	is the wave number;
$\alpha, \beta, \gamma, \lambda, \mu$, k	are the viscosity coefficients;
δ_{ij}	is the Kronecker delta symbol;
ε_{ijk}	is the Levi-Civita tensor density.

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OSCILLATIONS OF A VISCOELASTIC ROD TAKING THERMOMECHANICAL COUPLING INTO ACCOUNT

V. G. Karnaukhov and B. P. Gumenyuk

UDC 539.376

The effect of thermomechanical coupling on the forced longitudinal oscillations of a viscoelastic rod is investigated.

The wide use of viscoelastic materials in many areas of modern technology makes it important to investigate their behavior under different conditions. In this connection, it is of particular interest to study the interaction between the deformation and temperature fields, since viscoelastic materials have the ability to dissipate mechanical energy, and exhibit a considerable temperature dependence of their physicomechanical properties. Consideration of the thermomechanical coupling leads to nonlinearity in the mathematical formulation of the problem, and enables a number of extremely interesting nonlinear effects to be explained. These features of viscoelastic materials manifest themselves most clearly during cyclical deformation. It is shown in [1], using the example of the oscillations of a system with one degree of freedom (a large load on a viscoelastic spring) that over a certain range of variation of the excitation parameter the amplitude–frequency and temperature–frequency dependences are nonunique. These results were confirmed experimentally in [2]. A similar problem was considered in [3] where it was established that for periodic deformations two stable stationary states with different temperatures are possible. In this paper we investigate the effect of thermomechanical coupling on the dynamic behavior of a viscoelastic rod for forced longitudinal oscillations. Subcritical

Institute of Mechanics, Academy of Sciences of the Ukrainian SSR, Kiev.

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and critical thermal states are distinguished. We mean by a critical state the characteristic limiting values of the excitation parameter λ; when this is exceeded, thermal instability occurs (a sharp avalanchelike increase in temperature with time, the so-called thermal explosion), which leads to softening of the material. This can form the basis for a criterion for the exhaustion of the carrying capacity of components made of viscoelastic materials [4, 5].

Suppose a periodic force $\sigma_x = \sigma_0 \cos \omega t$ (σ_0 = const) is applied to the ends of a viscoelastic rod; the side surface and left end of the rod ($x = 0$) are thermally insulated, while the right end ($x = l$) is maintained at a constant temperature T_0. The rod material is assumed to be thermorheologically simple. The frequency and temperature dependence of the complex compliance for stretching has the form [6]

$$D^* = D_1 - iD_2 = (c_1 - ic_2)\,\omega^{\beta}\,(T - T_1)^{\gamma}. \tag{1}$$

The steady-state motion of such a rod in a stationary temperature field is described by the following nonlinear system of differential equations [6]:

$$\begin{aligned} &s_1'' + (1+\theta)^{\gamma}(b_1 s_1 + b_2 s_2) = 0,\\ &s_2'' + (1+\theta)^{\gamma}(b_1 s_2 - b_2 s_1) = 0,\\ &\theta'' + b_2 (1+\theta)^{\gamma}(s_1^2 + s_2^2) = 0 \end{aligned} \tag{2}$$

with the boundary conditions

$$\begin{aligned} &s_1 = s_0,\ s_2 = 0,\ \theta' = 0\ (\xi = 0),\\ &s_1 = s_0,\ s_2 = 0,\ \theta = 0\ (\xi = 1), \end{aligned} \tag{3}$$

where

$$s_{0,1,2} = \alpha\sigma_{0,1,2};\ \sigma = \sigma_1 + i\sigma_2;\ \alpha = (2\lambda_q \rho\omega T_2)^{-\frac{1}{2}};\ \xi = x/l;\ \theta = (T - T_0)/T_2;$$
$$T_2 = T_0 - T_1;\ b_{1,2} = c_{1,2}\rho l^2 \omega^{2+\beta} T_2^{\gamma}.$$

The problem of determining the critical thermal state of the rod can be formulated as the problem of finding the critical value of λ_* of the parameter λ, above which there are no solutions of the boundary-value problem (2) and (3).

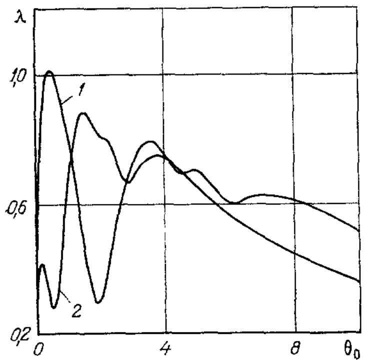
To solve Eqs. (2) and (3) we will use the finite-difference approach proposed and tested in [7]. The interval $0 \le \xi \le 1$ will be divided into N parts of length h at the points $\xi_j = jh$ ($j = 0, 1, \ldots, N$). The introduction of difference relations with a second order of approximation [8] leads to a system of 3N−3 nonlinear algebraic equations in the quantities $s_{1,2}(\xi_j)$ ($j = 1, 2, \ldots, N-1$), $\theta(\xi_j)$ ($j = 0, 2, 3, \ldots, N-1$). In this case $\theta(\xi_1) = \theta(\xi_0) - b_2 s_0^2 [1 + \theta(\xi_0)]^{\gamma} h^2/2$. The algebraic system obtained is solved by the method of steepest descent [9]. As a result we obtain the stress state and the temperature. To calculate the critical thermal state we investigated the dependence of the parameter

$$\lambda = \left(\frac{1}{2\lambda_q} c_2 l^2 \omega_0^{1+\beta} T_2^{\gamma-1}\right)^{\frac{1}{2}} \sigma_0 \tag{4}$$

on the maximum temperature $\theta_0 = \theta(0)$, where $\omega_0 = (\pi^2/c_1 \rho l^2 T_2^{\gamma})^{1/2+\beta}$. The highest value on the $\lambda(\theta_0)$ curve is the critical value λ_* of the parameter λ. The dependence $\lambda(\theta_0)$ is found by solving the above system of algebraic equations in which the quantity θ_0 is assigned, while the parameter λ is assumed to be required.

Numerical results obtained were for a rod of typical viscoelastic material with the following data [8]: ρ = 1214 kg/m^3, $c_1 = 4.43 \cdot 10^{-4}$ m^2/N, $c_2 = 1.56 \cdot 10^{-14}$ m^2/N, $\beta = -0.214$, $\gamma = 3.21$, $l = 0.0762$ m, $\lambda_q = 0.15$ W/m·deg, $T_0 = 18.3$°C, and $T_1 = -87.2$°C.

Figure 1 shows curves of $\lambda(\theta_0)$ for $\omega = 1.5 \cdot 10^3$ sec^{-1} (curve 1) and $\omega = 4 \cdot 10^3$ sec^{-1} (curve 2). We will indicate the extremal values λ^0 on the $\lambda(\theta_0)$ curves: λ_k^s ($k = 1, 2, 3, 4$) are the maxima, and λ_k^i ($k = 1, 2, 3$) are the minima. For curve 1, $\lambda_1^s > \lambda_2^s$, and for curve 2, $\lambda_1^s < \lambda_2^s$. The presence of several maxima on the curve confirms the existence of several stable states (the monotonically increasing parts) and unstable states (the monotonically decreasing parts). These states are defined by the points of intersection of the straight line λ = const with the $\lambda(\theta_0)$ curve. The realization of one state or another depends on the initial conditions of the nonstationary problem. In a number of situations a change from one state to another is possible. The case when the straight line λ = const lies above the $\lambda(\theta_0)$ curve, corresponds to thermal instability. The critical value λ_* of the parameter λ is the greatest of the values λ_k^s.



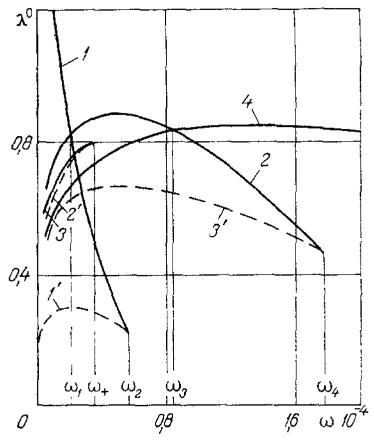


Fig. 1

Fig. 2

Fig. 1. Curves of $\lambda(\theta_0)$ for fixed values of the frequency: 1) $\omega = 1.5 \cdot 10^3$ sec^{-1}, 2) $\omega = 4 \cdot 10^3$ sec^{-1}.

Fig. 2. Frequency dependence of the extremal values of the $\lambda^0(\omega)$ curves; ω is in sec^{-1}.

Figure 2 shows curves of $\lambda^0(\omega)$; the continuous lines correspond to the $\lambda_k^s(\omega)$ curves, and the dashed lines correspond to the $\lambda_k^i(\omega)$ curves. Over the frequency range $0 < \omega \le \omega_+$ there are four stable states, for $\omega_+ < \omega \le \omega_2$ there are three stable states, for $\omega_2 < \omega \le \omega_4$ there are two stable states, and for $\omega > \omega_4$ there is one stable state. In the range $\omega_1 < \omega < \omega_2$ transition from the first stable state to the second is possible (with $\lambda > \lambda_1^s$), and in the range $\omega_3 < \omega < \omega_4$, transition from the second stable state to the fourth is possible (with $\lambda > \lambda_2^s$). In the range $0 = \omega \le \omega_1$, $\lambda_* = \lambda_1^s$, for $\omega_1 < \omega \le \omega_3$, $\lambda_* = \lambda_2^s$, and for $\omega > \omega_3$, $\lambda_* = \lambda_4$.

For the subcritical thermal states ($\lambda < \lambda_*$) the dependences of the temperature on the loading frequency were obtained. Curves of $\theta_0(\omega)$ are shown in Fig. 3a ($\lambda = 0.289$ – curve 1, $\lambda = 0.42$ – curve 2), 3b ($\lambda = 0.66$), and 3c ($\lambda = 0.84$). The continuous lines correspond to stable branches of the curves, and the dashed lines to unstable branches. Curve 1 in Fig. 3a is characteristic of the fact that the value $\lambda = 0.289$ lies in the range of variation of $\lambda_1^i(\omega)$. When ω increases the temperature increases slowly to the point I ($\omega = 5 \cdot 10^3$ sec^{-1}), from which a jump occurs to the point II on the second stable branch. When ω is increased further, motion occurs along the part II-III. When ω is reduced the temperature is determined by the point on the III-IV section; from point IV ($\omega = 3.3 \cdot 10^3$ sec^{-1}) a jump occurs to the point V on the first stable branch; then the part V–0 is obtained. The point of intersection of the straight line $\lambda = 0.289$ and curve 1 in Fig. 2 corresponds to the point I. The jump to point II occurs when the condition $\lambda_1^s < 0.289$ begins to be satisfied. The second point of intersection of the straight line $\lambda = 0.289$ with curve 1' in Fig. 2 corresponds to point IV. A jump to point V occurs as soon as the condition $\lambda_1^i > 0.289$ begins to be satisfied as ω decreases. If the initial condition of the nonstationary problem is such that the stationary temperature is described by a point on the VI-VII branch, which, like the III-IV branch corresponds to the second stable branch of the $\lambda(\theta_0)$ curves, then as ω increases, and reaches the point VII ($\omega = 1.15 \cdot 10^3$ sec^{-1}) a jump occurs to point VIII on the first stable branch. The first point of intersection of the straight line $\lambda = 0.289$ with curve 1' in Fig. 2 corresponds to points VII. A jump occurs to point VIII as soon as $\lambda_1^i > 0.289$ as ω increases.

For curve 2 in Fig. 3a it is essential that the straight line $\lambda = 0.42$ should lie above the $\lambda_1^i(\omega)$ curve. In this case, as ω increases, and when the condition $\lambda_1^s < 0.42$ is satisfied ($\omega = 4 \cdot 10^3$ sec^{-1}) a jump occurs from point IX to the first stable branch at the point X on the second stable branch XI-XII, but when ω decreases the reverse jump from the second stable branch to the first does not occur.

The $\theta_0(\omega)$ curve in Fig. 3b represents the case when the straight line λ = const intersects all the $\lambda^0(\omega)$ curves, with the exception of $\lambda_1^i(\omega)$. For this case the presence of isolated stable branches and two abrupt transitions from the lower branches to the higher branches is characteristic. The curve 0-I is the first stable branch, III-IV is the second, XIV-XV is the third, and the part X-XI and the curve V-VI is the fourth. When ω increases from 0 the branch 0-I is obtained. The point of intersection of the straight line $\lambda = 0.66$ with the $\lambda_1^s(\omega)$ curve corresponds to point I ($\omega = 2.75 \cdot 10^3$ sec^{-1}. When $\lambda_1^s < 0.66$ a jump occurs to point II. The part II-III is then obtained. The second point of intersection of the straight line $\lambda = 0.66$ with the $\lambda_2^s(\omega)$ curve corresponds to point III ($\omega = 1.36 \cdot 10^4$ sec^{-1}). When $\lambda_2^s < 0.66$ intersection occurs at the point IV. The part IV-V

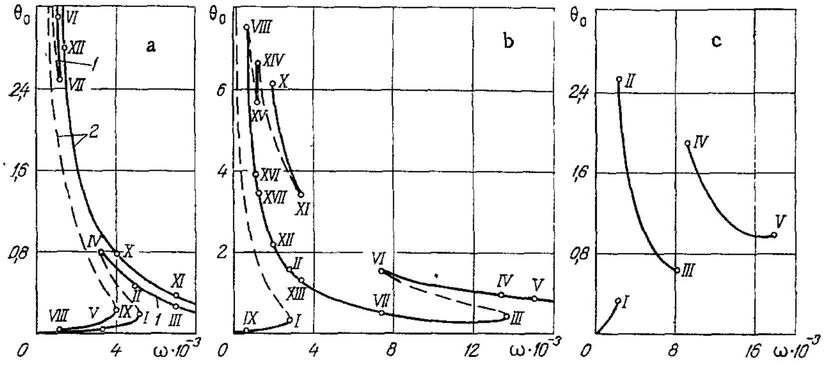


Fig. 3. Variation with frequency of the dimensionless temperature. a) 1 — $\lambda = 0.289$, 2 — $\lambda = 0.42$, b) $\lambda = 0.66$, c) $\lambda = 0.84$; ω, $\sec^{-1}$.

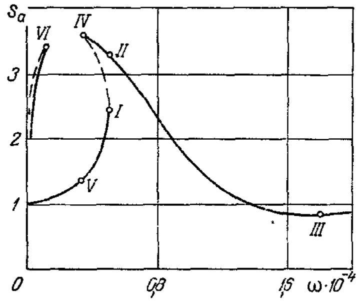


Fig. 4. Variation with frequency of the amplitude of the dimensionless stress at the middle point of the rod.

is then obtained. The part V-VI is obtained as ω decreases. The second point of intersection of the straight line $\lambda = 0.66$ with the $\lambda_3^i(\omega)$ curve corresponds to the point VI ($\omega = 7.5 \cdot 10^3 \sec^{-1}$). When $\lambda_3^i < 0.66$ a jump occurs to VII. The part VII-VIII is then obtained. The first point of intersection of the straight line $\lambda = 0.66$ with the $\lambda_2^s(\omega)$ curve corresponds to the point VIII ($\omega = 500 \sec^{-1}$). A jump occurs from point VIII (for $\lambda_2^s < 0.66$) to point IX. The point of intersection of the straight line $\lambda = 0.66$ with the $\lambda_4^s(\omega)$ curve and the first point of intersection of the straight line $\lambda = 0.66$ with the $\lambda_3^i(\omega)$ curve correspond to the points X ($\omega = 1.9 \cdot 10^3 \sec^{-1}$) and XI ($\omega = 3.3 \cdot 10^3 \sec^{-1}$). If the temperature is determined by points on the X-XI section, then from the point X, as ω decreases when and the condition $\lambda_4^s < 0.66$ is satisfied, and from the point XI, as ω increases and when the condition $\lambda_3^i > 0.66$ is satisfied, a jump occurs to the points XII and XIII, respectively. The points of intersection of the straight line $\lambda = 0.66$ with the $\lambda_3^s(\omega)$ and $\lambda_2^i(\omega)$ curves correspond to the points XIV ($\omega = 10^3 \sec^{-1}$) and XV ($\omega = 1.1 \cdot 10^3 \sec^{-1}$). If the temperature is determined by points on the XIV-XV section, then from the point XIV, as ω decreases, and when the condition $\lambda_3^s < 0.66$ is satisfied, and from the point XV as ω increases and when the condition $\lambda_2^i > 0.66$ is satisfied, a jump occurs to the points XVI and XVII, respectively.

If the straight line $\lambda = \text{const}$ is situated above all the $\lambda_k^i(\omega)$ curves, a transition from the low-temperature branches of the $\theta_0(\omega)$ curve to the high-temperature branches can occur, but the reverse transition is impossible. Figure 3c shows stable branches of the $\theta_0(\omega)$ curve for $\lambda = 0.84$. In this case they are isolated. The point of intersection of the straight line $\lambda = 0.84$ with the $\lambda_1^s(\omega)$ curve corresponds to the point I ($\omega_I = 2 \cdot 10^3 \sec^{-1}$), and the first and second points of intersection of the straight line $\lambda = 0.84$ with the $\lambda_1^s(\omega)$ curve corresponds to the points II ($\omega_{II} = 2.24 \cdot 10^3 \sec^{-1}$) and III ($\omega_{III} = 8.05 \cdot 10^3 \sec^{-1}$), and the first and second points of intersection of the straight line $\lambda = 0.84$ with the $\lambda_2^s(\omega)$ curve correspond to points IV ($\omega_{IV} = 9 \cdot 10^3 \sec^{-1}$) and V ($\omega_V = 1.76 \cdot 10^4 \sec^{-1}$). In the ranges $\omega_I < \omega < \omega_{II}$, $\omega_{III} < \omega < \omega_{IV}$ and $\omega > \omega_V$ thermal instability is observed.

The variation with frequency of the stress in the rod has the same form as the temperature variation. Figure 4 shows the frequency dependence of the amplitude $s_a = (s_1^2 + s_2^2)^{1/2}/s_0$ of the dimensionless stress at the middle point of the rod ($\xi = 0.5$) for $\lambda = 0.289$.

It follows from the above results that when thermomechanical coupling is taken into account a viscoelastic rod behaves as a nonlinear mechanical system with a soft type of characteristic.

NOTATION

x	is the coordinate along the rod;
l	is the length of the rod;
c_2, c_2, β, γ, T_1	are the constants of the material;
T	is the temperature;
$\sigma = \sigma_1 + i\sigma_2$	is the complex amplitude of the harmonic stress;
ρ	is the density;
λ_2	is the thermal conductivity;
ω	is the angular frequency;
ξ, θ, s_1, s_2	are the dimensionless coordinate, temperature, and stress components;
λ	is the dimensionless loading parameter.

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CALCULATION OF KINEMATIC COAGULATION OF AN AEROSOL IN A VARIABLE-SPEED GAS STREAM

I. B. Palatnik and A. K. Azhibekov

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A method is suggested for calculating the kinematic coagulation of drops in a variable-speed gas stream when they are broken up by the gas stream. The results of the calculation are compared with test data.

The problem of coagulation, particularly of colloids, under the action of Brownian motion was first analyzed by Smolukhovskii [1] for the case of an isodisperse distribution. The equations for the general case of coagulation with a continuous polydisperse distribution were analyzed by Müller [2] and Tunitskii [3].

Two approaches to the calculation of particle coagulation are known (see [4], for example). The first is based on the study of the evolution of the drop sizes of the fractions under consideration. This method, because of a certain analogy with classical hydrodynamics, received the name of the Lagrange method. The second is based on the determination of the numbers of particles of fixed sizes and is named the Euler method.

Henceforth we will analyze the problem of coagulation of a polydisperse system of particles by the Euler method.

When the particle spectrum is assigned in the form of a varying mass distribution function

$$dN = f(m, \tau)\, dm, \tag{1}$$

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